# Auxiliary Loop Antennas For AM Reception

Dipl.-Phys. Jochen Bauer

06/01/2014

**Abstract**

A common way of improving the reception of weak stations by an AM pocket radio with a relatively small build-in ferrite rod antenna is to place a large tuned loop antenna near it and tune this auxiliary loop antenna to the same reception frequency as the pocket radio. Obviously, this amounts to an inductive coupling between the tuned auxiliary loop antenna and the tuned build-in ferrite rod antenna and hence, this setup is similar to a tuned transformer. However, unlike the standard tuned transformer that is treated extensively in the literature, we are here dealing with a tuned transformer where the primary and the secondary side are driven synchronously by an incident electromagnetic wave. In this paper, we shall therefore re-derive some of the equations governing the tuned transformer and augment them to include the case of a synchronously driven secondary side. From the resulting equations, an expression for the optimal mutual inductive coupling factor between the auxiliary and build-in antennas is derived and it is shown that the optimal coupling depends on the voltage pickup ratio between the auxiliary and build-in antennas. It is also shown that the auxiliary loop antenna needs to meet certain performance criteria to actually improve reception. Furthermore, it is shown that reversing the direction of the build-in ferrite rod antenna with respect to the auxiliary loop antenna (turning the pocket radio by 180°) will have no effect on reception.

## Introduction and Overview

The performance of AM pocket radios with small built-in ferrite rod antennas is in most cases inferior to radios with larger internal loop antennas. Since pocket radios are typically not equipped with jacks to connect external antennas for improved reception, a very common workaround is to place the pocket radio inside a large tuned loop antenna with it's ferrite rod parallel to the loop's axis and exploit the inductive coupling between the built-in ferrite rod antenna and the outside loop as depicted in figure 1. The outside loop is tuned to the same reception frequency as the build-in antenna tank in the pocket receiver. This makes it possible to use large tuned loop antennas as auxiliary antennas for improved reception without having to tamper with the circuit of the pocket radio. Some practical examples of this setup can be found in [1].
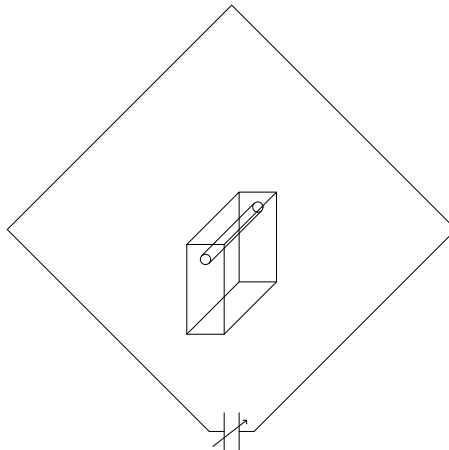
---

Figure 1: Radio with ferrite rod antenna in a large tuned loop antenna

Let us first take a look at this system from a qualitative point of view that will give us some basic understanding of what is going on: The auxiliary loop antenna will usually pick up a much larger driving voltage from the same incident electromagnetic wave than the built-in antenna. This driving voltage will cause a resonant current in the auxiliary loop antenna tank which will in turn induce an extra driving voltage into the built-in antenna tank. This "indirect driving" mechanism will therefore increase the overall driving voltage in the built-in antenna tank and improve reception. However, since the auxiliary loop antenna tank is also lossy, coupling it to the built-in antenna will at the same time increase losses in the built-in antenna tank, thereby reducing the resonant current in the built-in antenna tank.

Obviously, the presence of the auxiliary loop antenna has two effects on the built-in antenna tank that are countering each other. This will typically lead to the coupling factor between the auxiliary antenna and the built-in antenna to be of crucial importance for the actual improvement in reception. We therefore need to tackle the problem by a rigorous quantitative analysis.

## The Lossy Tuned Transformer Model

The setup of an auxiliary tuned loop antenna coupled inductively to the built-in antenna tank and tuned to the same resonant frequency is, of course, nothing else than a (lossy) tuned transformer with the auxiliary tuned loop being the primary and the built-in antenna tank being the secondary tuned circuit. However, unlike the standard tuned transformer used e.g. as an IF filter, this tuned transformer here is not only driven on the primary side (auxiliary loop antenna) but also on the secondary side since (built-in antenna tank) since an incident electromagnetic

wave will cause a driving voltage in both. We therefore need to augment the standard tuned transformer and allow for a driven secondary side. The resulting circuit is depicted in figure 2.

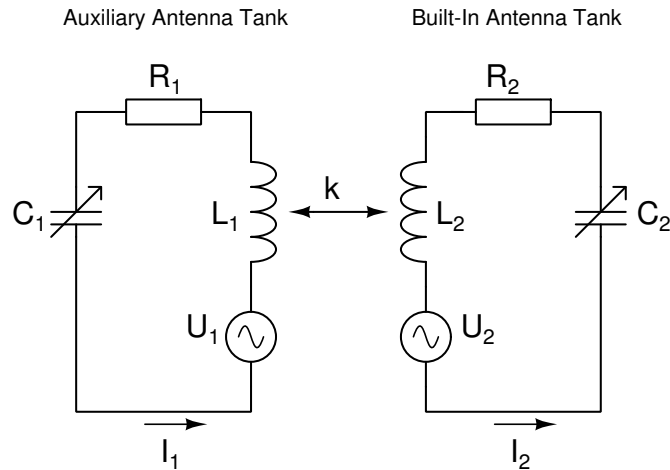Auxiliary Antenna Tank          Built-In Antenna Tank

Figure 2: Auxiliary and built-in antenna tanks as a lossy tuned transformer

Let us first take a look at the driving voltages: The auxiliary tuned loop antenna is driven by a voltage $U_1(t)$, while the built-in antenna tank is driven by a voltage $U_2(t)$. Since both driving voltages are caused by the time-dependent magnetic field of the same incident electromagnetic wave, they will have the same phase as long as the auxiliary loop antenna is much closer to the receiver than a quarter wavelength of the incident electromagnetic wave. For AM broadcast wavelengths of 200m and above, this condition will be satisfied for any reasonable setup.

The (real-valued) amplitudes $\hat{U}_1$ and $\hat{U}_2$ of the driving voltages are therefore linked by a real-valued parameter $\alpha$ and we have

$$\hat{U}_2 = \alpha \hat{U}_1$$

This describes the driving voltage $\hat{U}_2$ occurring in the built-in antenna in terms of the driving voltage $\hat{U}_1$ occurring in the auxiliary loop antenna. In typical setups, $\alpha$ will satisfy $|\alpha| < 1$ since the large auxiliary antenna will have a higher voltage pickup than the built-in ferrite rod antenna, but the theory that is going to be developed also allows for $|\alpha| \geq 1$. The sign of $\alpha$ accounts for the relative orientation of the auxiliary loop and the coil around the ferrite rod. Rotating the pocket receiver by 180° relative to the loop will alter the sign of $\alpha$.

Let us now turn to the other components of the circuit. The inductance of the auxiliary loop is represented by $L_1$ while $L_2$ is the inductance of the built-in ferrite rod antenna. These inductors are coupled inductively by the mutual

coupling factor $k$. The losses in the auxiliary loop and built-in ferrite rod antenna tanks are modeled by a series loss resistance $R_1$, respectively $R_2$. that is related to the Q-factor of the tank by [2]

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{1}$$

The voltages and currents in this circuit are easily calculated using complex phasors. For our purposes here, we are interested in the absolute value $|\hat{I}_2|$ of the current amplitude in the built-in ferrite rod antenna tank as a function of the circuit parameters and the driving voltage amplitude $\hat{U}_1$ picked up by the auxiliary loop antenna [1] provided that both antenna tanks are tuned to the same reception frequency

$$\omega_0 = \sqrt{\frac{1}{L_1 C_1}} = \sqrt{\frac{1}{L_2 C_2}}$$

It turns out that $|\hat{I}_2|$ is given by

$$\boxed{|\hat{I}_2| = \frac{\sqrt{\frac{L_2}{R_1^2 C_1}k^2 + \alpha^2}}{R_2 + \frac{L_2}{R_1 C_1}k^2}\,\hat{U}_1}$$

(A detailed derivation of the above expression is given in **appendix A**.) The quantity $|\hat{I}_2|$ is an immediate measure of the reception strength of a given station in the pocket radio. As a first result, we notice that $|\hat{I}_2|$ does not depend on the sign of $\alpha$. In practice: Rotating the pocket receiver by 180° relative to the loop does not alter the reception strength for a given station. The reason for this can be seen in appendix A: The indirect driving voltage via the auxiliary loop tank and the voltage picked up by the built-in ferrite rod antenna directly have a relative phase shift of 90° if both tanks are tuned to the same resonant frequency.

## Optimal Coupling And Improvement Of Reception

At this point, let us get an idea of how a typical curve of $|\hat{I}_2|$ as a function of the coupling factor $k$ between the auxiliary loop and the built-in ferrite rod antenna looks like. Also, for comparison, we would like to know what $|\hat{I}_2|$ would be without the presence of the auxiliary antenna. This is quite easy to tell since at $k = 0$ the influence of the auxiliary antenna vanishes and hence in a plot of $|\hat{I}_2|$ over $k$ the reception strength without the auxiliary antenna is marked by the intersection of the curve with the vertical axis at $k = 0$.

---

[1]We could, of course, also have used the voltage amplitude $\hat{U}_2$ picked up by the built-in ferrite rod antenna instead.

In order to plot $|\hat{I}_2|$ versus the coupling factor $k$, we need to choose reasonable values for the remaining parameters. Our choice shall be $L_1 = L_2 = 300\mu H$ and $C_1 = C_2 = 200\text{pF}$, resulting in a resonant/reception frequency of approximately 650kHz, as well as $R_1 = 12\Omega$ and $R_2 = 24\Omega$, amounting to Q-factors of $Q_1 \approx 100$ and $Q_2 \approx 50$ for the auxiliary loop antenna tank, respectively the built-in ferrite rod antenna tank. Also, we shall set $\hat{U}_1 = 1\text{mV}$ and choose $\alpha = 0.1$, i.e. the voltage pickup of the auxiliary loop antenna is ten times higher than the voltage pickup of the build-in ferrite rod antenna. The resulting curve is shown in figure 3.
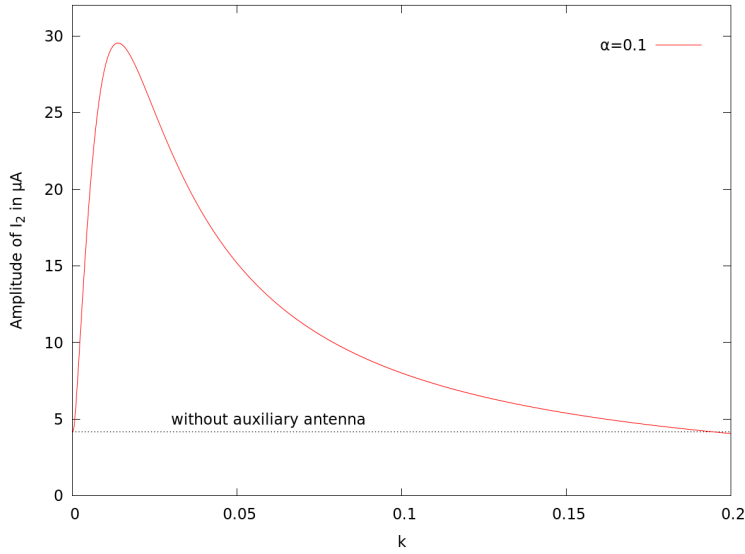


Figure 3: Build-in antenna tank current versus coupling factor

As hinted in the previous section, there is an optimal coupling factor $k_{\text{opt}}$ for which the improvement in reception is maximized. In practice, one would try to determine if the maximum coupling factor that occurs when the pocket receiver is at the center of the loop is already beyond the optimal coupling factor by moving the pocket receiver along the loop's axis away from the loop, thereby decreasing the coupling factor. Finding a theoretical expression for the optimal coupling factor is a matter of simple calculus: The maximum of the $|\hat{I}_2|$ curve as a function of $k$ occurs when

$$\frac{\partial |\hat{I}_2|}{\partial k} = 0$$

This condition can be satisfied by

$$k_{\text{opt}} = R_1 \sqrt{\frac{C_1}{L_2}} \sqrt{\frac{R_2}{R_1} - 2\alpha^2} \tag{2}$$

as long as

$$\boxed{\alpha^2 < \frac{R_2}{2R_1}}$$

We notice that $|\hat{I}_2|$ reaches it's maximum at a non-zero $k$ only if the above condition is fulfilled. Otherwise, $|\hat{I}_2|$ reaches it's maximum at $k = 0$, i.e. when the auxiliary loop antenna is not present at all. This behavior is illustrated in figure 4 by plotting some $|\hat{I}_2|$ over $k$ curves for increasing values of $\alpha$ with all other parameters being the same as in the previous example.
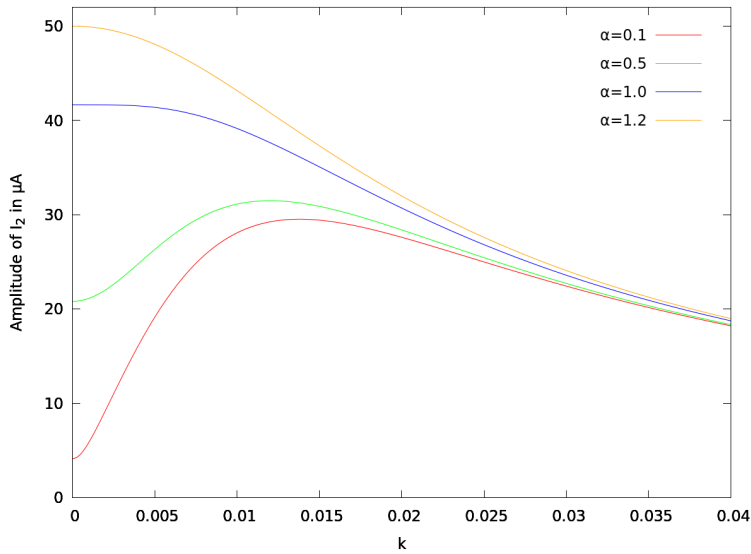


Figure 4: Build-in antenna tank current for different voltage pickup ratios

In practice, if the voltage pickup of the auxiliary antenna and/or it's Q-factor is not sufficiently high compared to the voltage pickup and Q-factor of the built-in antenna, the auxiliary antenna is not only useless but will even worsen reception.

A little more insight into the optimal coupling factor $k_{\mathrm{opt}}$ can be attained by using equation (1) to rewrite the expression for $k_{\mathrm{opt}}$ from equation (2) in terms of the Q-factors $Q_1$ and $Q_2$ of the antenna tanks and their inductance ratio $L_1/L_2$. The resulting expression is:

$$k_{\mathrm{opt}} = \sqrt{\frac{1}{Q_1 Q_2} - \alpha^2 \frac{2}{Q_1^2} \frac{L_1}{L_2}} \ , \quad \text{for} \quad \alpha^2 < \frac{1}{2} \frac{Q_1}{Q_2} \frac{L_2}{L_1}$$

It now becomes obvious that for $\alpha = 0$ the optimal coupling factor is

$$k_{\text{opt}} = \sqrt{\frac{1}{Q_1 Q_2}}$$

which is the critical coupling factor of the two antenna tanks [3]. For small values of $\alpha$ (i.e. the voltage pickup of the auxiliary loop antenna is much higher than the voltage pickup of the built-in ferrite rod antenna) the optimal coupling is therefore approximately critical. With $\alpha$ increasing, the optimal coupling factor is starting to drop into the sub critical coupling range.

## Appendix A: Derivation Of The Previously Used Equations

Using Kirchhoff's loop rule on the auxiliary loop and built-in ferrite rod antenna tanks, we find that

$$R_1 I_1(t) + \frac{1}{C_1} q_1(t) + U_{L1}(t) = U_1(t) \tag{3}$$

and

$$R_2 I_2(t) + \frac{1}{C_2} q_2(t) + U_{L2}(t) = U_2(t) \tag{4}$$

were $U_{L1}$ is the voltage at the terminals of the loop coil, $U_{L2}$ is the voltage at the terminals of the ferrite rod coil and $q_{1/2}$ denotes the charge of the respective capacitors [2] Using the mutual inductance $M$ [4] between the auxiliary antenna coil and the built-in ferrite rod coil we obtain

$$U_{L1}(t) = L_1 \dot{I}_1(t) + M \dot{I}_2(t) \tag{5}$$

and

$$U_{L2}(t) = L_2 \dot{I}_2(t) + M \dot{I}_1(t) \tag{6}$$

The mutual inductance $M$ is related to the mutual coupling factor $k$ by [4]

$$M = k\sqrt{L_1 L_2} \tag{7}$$

Since we are interested in the driven solutions of the system for sinusoidal driving voltages, we can introduce the complex phasors [3]

$$U_1(t) = \hat{U}_1 e^{j\omega t} \quad U_2(t) = \hat{U}_2 e^{j\omega t} \quad I_1(t) = \hat{I}_1 e^{j\omega t} \quad I_2(t) = \hat{I}_2 e^{j\omega t}$$

and a helper phasor

---

[2]We have used $q$ here instead of the more common capital $Q$ to avoid confusion with the Q-factors of the tanks.

[3]Note that the driving voltage amplitudes $\hat{U}_1$ and $\hat{U}_2$ are real-valued.

---

$$U_{L2}(t) = \hat{U}_{L2}e^{i\omega t}$$

Substituting equation (5) into equation (3) and differentiating the resulting expression with respect to time yields

$$R_1\dot{I}_1(t) + \frac{1}{C_1}I_1(t) + L_1\ddot{I}_1(t) + M\ddot{I}_2(t) = \dot{U}_1(t)$$

Using the appropriate phasors in the above equation and solving the resulting expression for $\hat{I}_1$ then gives

$$\hat{I}_1 = \frac{\omega^2 M\hat{I}_2 + j\omega\hat{U}_1}{j\omega R_1 + \frac{1}{C_1} - \omega^2 L_1} \tag{8}$$

Substituting phasors into equation (6) leads to

$$\hat{U}_{L2} = j\omega L_2\hat{I}_2 + j\omega M\hat{I}_1$$

and plugging in $\hat{I}_1$ from equation (8) we arrive at

$$\hat{U}_{L2} = \left(j\omega L_2 + Z_+\right)\hat{I}_2 - \hat{U}_d \tag{9}$$

with

$$Z_+ = \frac{R_1 M^2\omega^2}{\left(\frac{1}{\omega C_1} - \omega L_1\right)^2 + R_1^2} + j\frac{\left(\frac{1}{\omega C_1} - \omega L_1\right)M^2\omega^2}{\left(\frac{1}{\omega C_1} - \omega L_1\right)^2 + R_1^2}$$

and

$$\hat{U}_d = \frac{\hat{U}_1 M\omega}{\left(\frac{1}{\omega C_1} - \omega L_1\right)^2 + R_1^2}\left(\frac{1}{\omega C_1} - \omega L_1 - jR_1\right)$$

Here, $Z_+$ can be identified as the complex impedance that is reflected from the auxiliary loop antenna tank into to built-in ferrite rod antenna tank, while $\hat{U}_d$ is the complex amplitude of the indirect driving voltage entering the ferrite rod antenna tank via the auxiliary loop antenna tank. Differentiating equation (4) with respect to time yields

$$\dot{U}_{L2}(t) + R_2\dot{I}_2(t) + \frac{1}{C_2}I_2(t) = \dot{U}_2(t)$$

Using the appropriate phasors in the above equation and plugging in $\hat{U}_{L2}$ from equation (9) as well as using $\hat{U}_2 = \alpha\hat{U}_1$ and solving the resulting expression for $\hat{I}_2$, we obtain

---

$$\hat{I}_2 = \frac{\hat{U}_d + \alpha \hat{U}_1}{Z_2} \tag{10}$$

with

$$Z_2 = j\omega L_2 + \frac{1}{j\omega C_2} + R_2 + Z_+$$

Since both tanks are tuned to a common resonant frequency

$$\omega = \omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}$$

equation (10) then finally simplifies to

$$\hat{I}_2 = \frac{-j\frac{k}{R_1}\sqrt{\frac{L_2}{C_1}} + \alpha}{R_2 + \frac{L_2}{R_1 C_1} k^2} \hat{U}_1 \tag{11}$$

where we have switched from the mutual inductance $M$ between $L_1$ and $L_2$ to their mutual coupling factor $k$ as given by equation (7). From this result, the absolute value of $\hat{I}_2$ as used in the main text is readily obtained.

# References

[1] http://www.radiomuseum.org
    /forum/verbesserung_radio_empfang_radioempfangsverbesserungen.html

[2] http://en.wikipedia.org/wiki/Q_factor

[3] F. Langford-Smith (Editor), *Radiotron Designer's Handbook 4th Edition*, 1952

[4] http://en.wikipedia.org/wiki/Inductance