We'll start with a complete version of the circuit as given in diagram 1 that includes all currents.

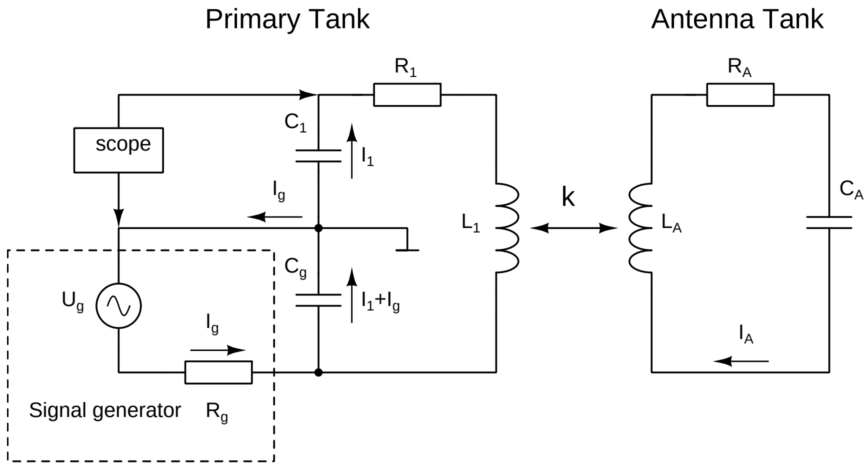

Figure 1: Detailed schematic including currents

From Kirchhoff's circuit laws we can deduce the following equations for the voltage phasors at the components

$$\begin{aligned} U_{R1} + U_{L1} + U_{C1} + U_{Cg} &= 0 \\ U_{Rg} + U_{Cg} &= U_g \\ U_{LA} + U_{RA} + U_{CA} &= 0 \end{aligned}$$

Using Ohm's law for complex impedances along with [1], these equation can be written as

$$R_1 I_1 + j\omega L_1 I_1 - j\omega M I_A + \frac{1}{j\omega C_1} I_1 + \frac{1}{j\omega C_g}(I_1 + I_g) = 0 \tag{1}$$

$$R_g I_g + \frac{1}{j\omega C_g}(I_1 + I_g) = U_g \tag{2}$$

$$j\omega L_A I_A - j\omega M I_1 + R_A I_A + \frac{1}{j\omega C_A} I_A = 0 \tag{3}$$

Where $M = k\sqrt{L_1 L_A}$ is the mutual inductance of the two inductors L_1 and L_A having an inductive coupling factor of k. Solving equation (2) for I_g and equation (3) for I_A and plugging the results into equation (1) yields

$$Z_1 I_1 = U_d \tag{4}$$

with

$$Z_1 = R_1 + j\omega L_1 - j\frac{1}{\omega C_1} - j\frac{1}{\omega C_g} + \frac{\left(\frac{1}{\omega C_g}\right)^2}{R_g - j\frac{1}{\omega C_g}} + \frac{(\omega M)^2}{R_{A} + j\left(\omega L_{A} - \frac{1}{\omega C_A}\right)} \tag{5}$$

and

$$U_d = \frac{j\frac{1}{\omega C_g}}{R_g - j\frac{1}{\omega C_g}} U_g \tag{6}$$

Obviously, U_d is the driving voltage of the primary tank resulting from the generator (open circuit) voltage U_g and Z_1 is the (series) Impedance that is seen by this driving voltage, resulting in a current I_1 in the primary tank.

Since C_g is chosen so that $C_g \gg C_1$ and $R_g \gg 1/\omega C_g$, equations (5) and (6) can be simplified to yield

$$Z_1 = R_1 + \frac{1}{R_g}\left(\frac{1}{\omega C_g}\right)^2 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) + \frac{(\omega M)^2\left(R_{A} - j\left(\omega L_{A} - \frac{1}{\omega C_A}\right)\right)}{R_{A}^2 + \left(\omega L_{A} - \frac{1}{\omega C_A}\right)^2} \tag{7}$$

and

$$U_d = \frac{j}{\omega C_g R_g} U_g \tag{8}$$

From equation (7) we can identify

$$Z_d = \frac{1}{R_g}\left(\frac{1}{\omega C_g}\right)^2$$

as the additional series loss resistance introduced into the primary tank by connecting a signal generator with an output impedance of R_g. Obviously, choosing a large shunt capacitor C_g helps to prevent the signal generator from putting to much extra load on the tank.

Also, we can immediately identify

$$Z_{A1} = \frac{(\omega M)^2\left(R_{A} - j\left(\omega L_{A} - \frac{1}{\omega C_A}\right)\right)}{R_{A}^2 + \left(\omega L_{A} - \frac{1}{\omega C_A}\right)^2}$$

as the impedance reflected from the secondary (antenna) tank into the primary tank. By looking a the above expression, it becomes obvious that when the resonant frequency of the secondary tank is above the resonant frequency of the primary tank, the reactance reflected into the primary tank is positive, i.e. the resonance peak of the primary tank is pushed towards lower frequencies as the inductive coupling factor is increased. The opposite applies if the resonant frequency of the secondary tank is below the resonant frequency of the primary tank.

We can now use equation (4) along with equations (7) and (8) to visualize the frequency response curve of the primary tank if the resonant frequencies of the primary and secondary tank are equal. The output of the primary tank is the voltage U_{C1} at the capacitor C_1 that is given by

$$U_{C1} = Z_{C1} I_1 = \frac{1}{j\omega C_1} \frac{U_d}{Z_1}$$

We can now plot the frequency response curve $|U_{C1}(\omega)|$. This has been done in diagram 2 for different inductive coupling factors k. The remaining parameters have been set as follows: $R_1 = 10\Omega$, $L_1 = 1200\mu\text{H}$, $C_1 = 60\text{pF}$, $C_g = 60\text{nF}$, $R_A = 150\Omega$, $L_A = 1320\mu\text{H}$, $C_A = 54.5\text{pF}$, $U_g = 1\text{V}$ and $R_g = 50\Omega$.

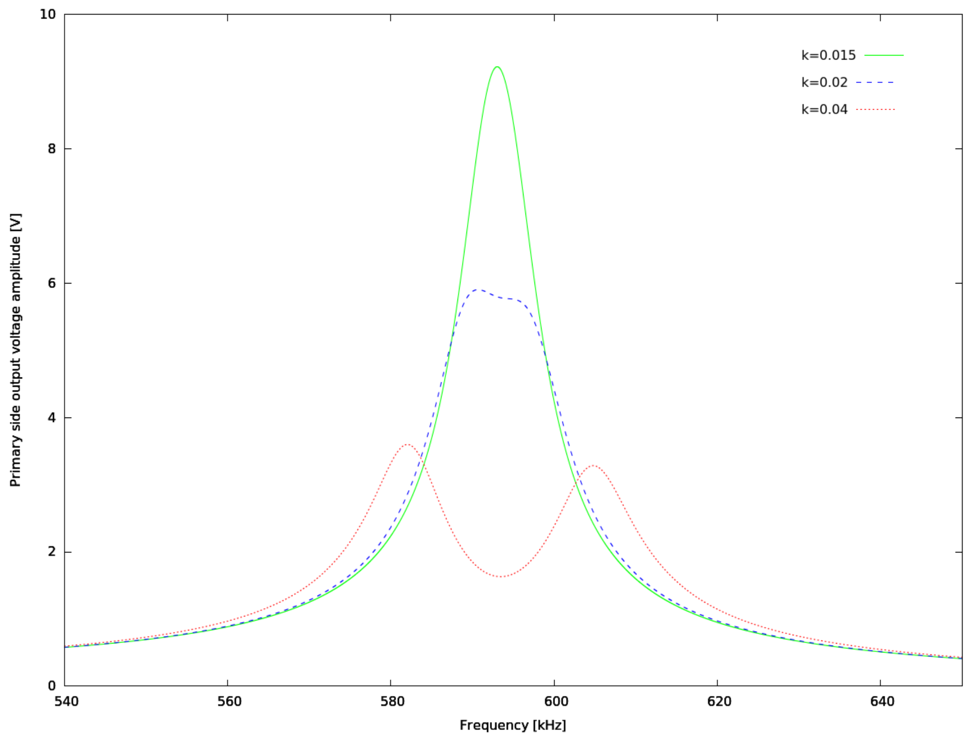

Figure 2: Frequency response curve of the primary side

Note that the slight lopsidedness of the frequency response curve occurs due to the fact that U_d and Z_{C1} both exhibit a $1/\omega$ dependency. Obviously, if the resonant frequency of the tanks are equal, increasing the inductive coupling factor will deform the single peak (unperturbed) frequency response curve of the primary side into the well-known double humped curve with the center dip located where the single resonance peak used to be.

Finally, we can use equations (7) and (8) to derive an expression for $|U_{C1}|$ at the center dip (equal resonant frequency ω_0 of both tanks). Setting

$$\omega_0 L_1 = \frac{1}{\omega_0 C_1} \qquad \text{and} \qquad \omega_0 L_A = \frac{1}{\omega_0 C_A}$$

in the previous equations and using $M = k\sqrt{L_1 L_A}$ immediately gives

$$|U_{C1}(\omega_0)| = \frac{1}{\omega_0 C_1} \cdot \frac{U_g}{\omega_0 C_g R_g \left(R_1 + \frac{1}{(\omega_0 C_g)^2 R_g} + \frac{k^2}{R_A} \omega_0^2 L_1 L_A \right)}$$

From this equation it becomes obvious that for a given, sufficiently large inductive coupling factor k the center dip will get deeper as the secondary (antenna) tank loss resistance R_A decreases.

References

[1] http://en.wikipedia.org/wiki/Inductance

[2] http://en.wikipedia.org/wiki/Kirchhoff's_circuit_laws