Tuned Circuits With Inductive Feedback - A Time Domain Approach

Dipl.-Phys. Jochen Bauer

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Abstract

In this paper, lossy tuned circuits with feedback provided inductively by a so-called "tickler coil" used as regenerative amplifiers or oscillators are analyzed in the time domain using differential equations. It is shown, that for the most important cases, the changes in the behavior of the tuned circuit due to the application of feedback can readily be seen from the structure of the resulting differential equations without the need to actually solve them. It turns out that providing feedback by a tickler coil placed in the plate (resp. drain) circuit of a pentode or field-effect transistor will alter the virtual loss resistance of the tuned circuit as desired while leaving the other parameters of the tuned circuit as they are. This applies to any voltage controlled current source driving the tickler coil. However, if the pentode or field-effect transistor is used in a voltage amplifier circuit with the tickler coil connected to the output, this setup will only provide proper feedback if the output impedance of the voltage amplifier is above a certain minimum. This applies to any voltage amplifier with the tickler coil connected to it's output. It is in contrast to many other applications where a voltage amplifier needs to have an output impedance as low as possible.

This paper is organized as follows: First, the general differential equation governing a lossy tuned circuit with inductive feedback is presented. A setup where the tickler coil is placed in the plate circuit of a pentode and thus driven by a voltage controlled current source is analyzed. Then, the setup is changed to the tickler coil being connected to the output of a voltage amplifier and an analysis is carried out for this circuit. The results for both setups are then summarized. For reference purposes, a summary of variables and constants used in this paper is given in **appendix A**.

Lossy Tuned Circuits With Inductive Feedback

A lossy tuned circuit can be modeled by introducing a resistance R connected in series with the inductor and capacitor of the tuned circuit. The losses in the components of the tuned circuit are summed up into this loss resistor. Feedback is applied to such a tuned circuit in order to partially (regenerative amplifier) or fully (oscillator) compensate the losses. One of the most common methods of providing feedback to a tuned circuit is using a feedback coil that is inductively coupled to the coil of the tuned circuit. This feedback coil is often called a "tickler coil". The current in $I_{\mathrm{f}}(t)$ the tickler coil needs to be related to the voltage $U_C(t)$ at the capacitor. This setup is depicted in figure 1.

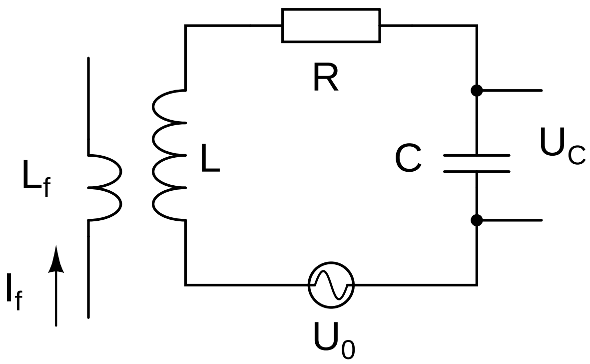

Figure 1: Basic principle of inductive feedback

The driving voltage $U_0(t)$ is the input voltage from the antenna or previous RF stage. Note that for oscillator circuits $U_0(t) \equiv 0$. The basic idea is to apply feedback in such a way that the circuit behaves as if it had a loss resistance $\tilde{R}$ lower than it's actual physical loss resistance R. The resistance $\tilde{R}$ can be dubbed "virtual loss resistance." For a general introduction into the topic, the reader is referred to [1].

The equations governing the lossy tuned circuit with inductive feedback have been derived in [1]. The voltages at the inductor L, the loss resistance R, the capacitor C and the feedback voltage $U_{\mathrm{f}}(t)$ sum up to the driving voltage $U_0(t)$ of the lossy tuned circuit. Hence,

$$U_L(t) + U_R(t) + U_C(t) + U_{\mathrm{f}}(t) = U_0(t)$$

The feedback voltage $U_{\mathrm{f}}(t)$ induced into the coil of tuned circuit by the tickler coil is [1]

$$U_{\mathrm{f}}(t) = -k\frac{n}{n_{\mathrm{f}}} L_{\mathrm{f}} \dot{I}_{\mathrm{f}}(t)$$

where $-1 \leq k \leq 1$ is the strength and direction of the inductive coupling between the tuned circuit's coil and the tickler coil, n is the number of turns of the tuned circuit's coil, n_{f} is the number of turns of the tickler coil and L_{f} is the inductance of the tickler coil. $\dot{I}_{\mathrm{f}}(t)$ is the derivative of the current $I_{\mathrm{f}}(t)$ in the tickler coil with respect to time. The resulting differential equation for the current $I(t)$ in the tuned circuit is [1]

$$\boxed{L\ddot{I}(t) + R\dot{I}(t) + \frac{1}{C}I(t) - k\frac{n}{n_{\mathrm{f}}}L_{\mathrm{f}}\ddot{I}_{\mathrm{f}}(t) = \dot{U}_0(t)} \tag{1}$$

The above equation shall be our starting point for further investigations. While in [1] a generic "black box" feedback device was used, we shall here focus on practical circuits using vacuum tubes. We will prefer pentodes over triodes because their low plate to grid capacitance will avoid undesired capacitive plate to grid feedback interfering with the inductive feedback by the tickler coil. In the presented circuits, the pentodes can easily be replaced by field-effect transistors, making all considerations done below immediately applicable to modern day circuits.

Tickler Coil In The Plate Circuit

First, we will look at a setup where the tickler coil is placed in the plate circuit of a pentode as depicted in figure 2.

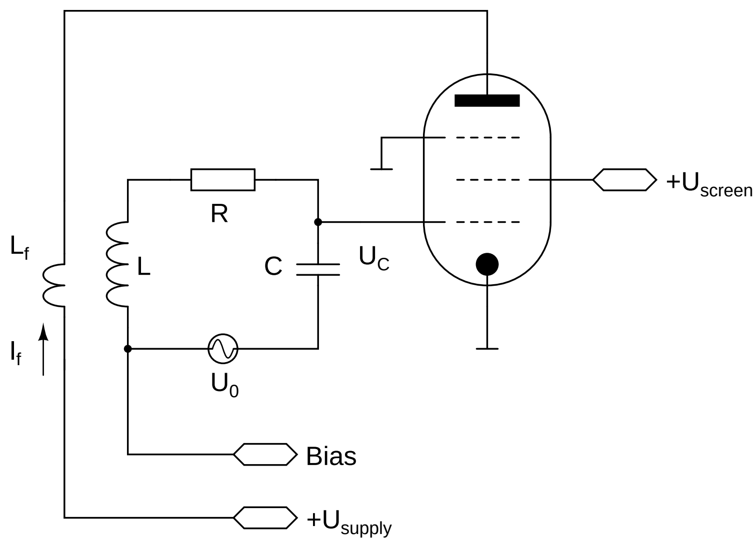

Figure 2: Feedback coil in the plate circuit

Here, the current in the tickler coil has an inevitable DC component that is the quiescent current of the pentode stage. For air core coils, this poses no problem. However, depending on the quiescent current, ferrite cores might be driven into magnetic saturation by the quiescent DC current. Since the pentode behaves much like a current source with the plate current being controlled by the grid voltage and being largely independent of the plate voltage, an AC equivalent circuit can be modeled using a current source controlled by the capacitor voltage of the tuned circuit as depicted in figure 3.

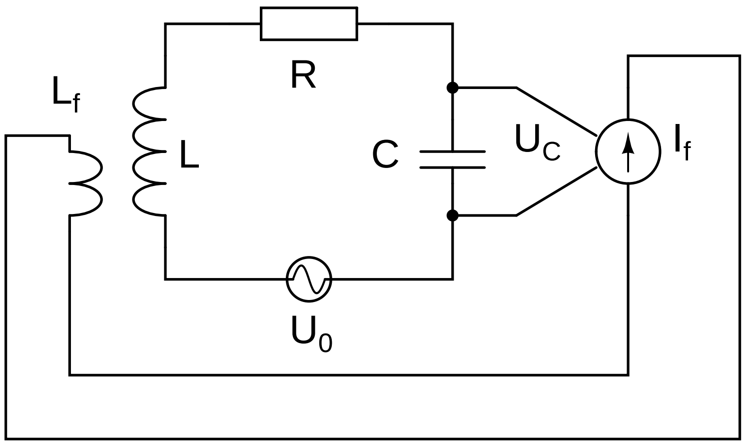

Figure 3: Feedback coil driven by a current source

If the amplitude of the AC voltage $U_C(t)$ at the pentode grid is sufficiently small, the dependence of the AC plate current on the AC grid voltage is approximately linear. We therefore can assume that

$$I_{\mathrm{f}}(t) = \beta U_C(t)$$

This is of course nothing else than the "black box" feedback mechanism that was used in [1] where β is the transconductance of the pentode at a given quiescent. Substituting the above equation into (1) and sorting the left hand side by $I(t)$ and it's derivatives $\dot{I}(t)$ and $\ddot{I}(t)$, we obtain

$$\boxed{L\ddot{I}(t) + \left(R - \frac{nL_{\mathrm{f}}\beta k}{n_{\mathrm{f}}C}\right)\dot{I}(t) + \frac{1}{C}I(t) = \dot{U}_0(t)}$$

Where we have used that $U_C(t) = Q(t)/C$ with $Q(t)$ being the charge of the capacitor of the tuned circuit. The physical loss resistance R has been replaced by a virtual loss resistance

$$\tilde{R} = R - \frac{nL_{\mathrm{f}}\beta k}{n_{\mathrm{f}}C}$$

that is lower than the physical loss resistance provided that $k\beta > 0$. This is the desired effect of the employed feedback in regenerative amplifiers ($R > \tilde{R} > 0$) and oscillators ($\tilde{R} \leq 0$) Note that in this case the physical inductance L and physical capacitance C of the tuned circuit do not get replaced. Hence, there is no undesired "side-effect" of the feedback in a sense that other parameters of the tuned circuit besides R are altered.

Tickler Coil Connected To A Voltage Amplifier

In this setup, an (ohmic) plate resistor R_{plate} is connected between the plate and the positive supply rail voltage and the pentode stage is used as a voltage amplifier with it's output connected to the tickler coil. Since the plate is at a high DC voltage, a coupling capacitor C_{f} needs to be placed in front of the tickler coil. The resulting circuit is shown in figure 4

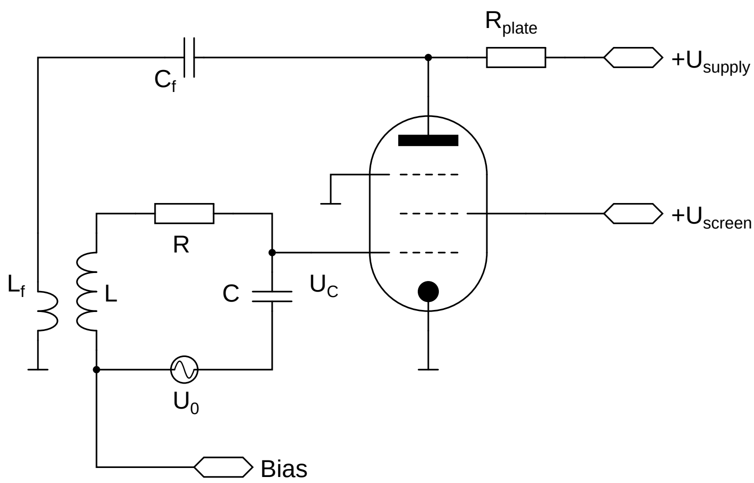

Figure 4: Feedback with a resistive plate load

By treating this pentode stage as a voltage amplifier with an open circuit amplification factor of α and an output impedance of R_{out} an AC equivalent circuit depicted in figure 5 can be modeled.

Again, if the amplitude of the AC voltage $U_C(t)$ at the pentode grid is sufficiently small, the dependence of the AC plate voltage $U_{\mathrm{out}}(t)$ on the AC grid voltage is approximately linear. We therefore have

$$U_{\mathrm{out}}(t) = \alpha U_C(t)$$

where α is R_{plate} times the transconductance of the pentode at a given quiescent. Since the internal AC plate resistance of a pentode is usually much higher than

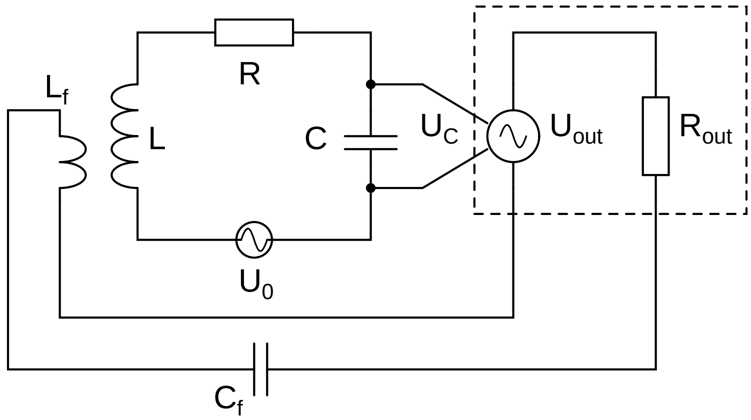

Figure 5: Feedback coil driven by a voltage source

the resistance R_{plate} of the external plate resistor, the output impedance R_{out} of this pentode stage is roughly equal to R_{plate}. We shall, however, continue to use the parameters α and R_{out} because this will make the following considerations applicable to *any* voltage amplifier with amplification factor α and output impedance R_{out}.

In the feedback circuit, the voltages at the tickler coil, the coupling capacitor and the output impedance equivalent resistor will sum up to the open circuit output voltage $U_{\text{out}}(t)$. This approach will lead to a set of two mutually coupled ordinary differential equations. Since the ordinary differential equations involved are linear, a closed form solution can be calculated [2]. However, the resulting expressions are quite cumbersome. Here, we will therefore look at three obvious special cases that will give us a qualitative understanding of the behavior of the circuit without the need to actually solve the resulting differential equations. (Although in these special cases the resulting expressions would be rather simple).

Dominant Output Impedance

Let us first look at the case where the output impedance R_{out} of the voltage amplifier is much bigger than the absolute value of the reactance of the coupling capacitor and the tickler coil ($R_{\text{out}} \gg 1/\omega C_{\text{f}}$ and $R_{\text{out}} \gg \omega L_{\text{f}}$). Let us further assume that the voltage induced into the tickler coil from the tuned circuit coil (bear in mind that the inductive coupling between those two coils is mutual) is much smaller than the AC output voltage $U_{\text{out}}(t)$ of the amplifier. The current $I_{\text{f}}(t)$ in the tickler coil is then given by

$$I_{\text{f}}(t) = \frac{U_{\text{out}}(t)}{R_{\text{out}}}$$

Using $U_{\text{out}}(t) = \alpha U_C(t)$ and $U_C(t) = Q(t)/C$, where $Q(t)$ is the charge of the capacitor of the tuned circuit in the above equation, we obtain

$$I_{\text{f}}(t) = \frac{\alpha Q(t)}{R_{\text{out}} C}$$

Differentiating the above expression twice with respect to time, we arrive at

$$\ddot{I}_{\text{f}}(t) = \frac{\alpha \dot{I}(t)}{R_{\text{out}} C}$$

which can readily be substituted into (1). Sorting the left hand side of the resulting equation by $I(t)$ and it's derivatives then finally yields

$$\boxed{L\ddot{I}(t) + \left(R - \frac{nL_{\text{f}}\alpha k}{n_{\text{f}} R_{\text{out}} C}\right)\dot{I}(t) + \frac{1}{C}I(t) = \dot{U}_0(t)}$$

Again, the physical loss resistance R has been replaced by a virtual loss resistance

$$\tilde{R} = R - \frac{nL_{\text{f}}\alpha k}{n_{\text{f}} R_{\text{out}} C}$$

that is lower than the physical loss resistance provided that $\alpha k > 0$. Again, this is the desired effect of the applied feedback. It is worth noting that since $\alpha = \beta R_{\text{plate}}$ and $R_{\text{out}} \approx R_{\text{plate}}$ for this pentode stage, the resulting virtual loss resistance is the same as in the previous circuit (figure 2) with the tickler coil in the plate circuit.

Dominant Coupling Capacitor Reactance

Let us now turn to the case where the absolute value of the reactance of the coupling capacitor is much bigger than the output impedance of the voltage amplifier and the absolute value of the reactance of the tickler coil ($1/\omega C_{\text{f}} \gg R_{\text{out}}$ and $1/\omega C_{\text{f}} \gg \omega L_{\text{f}}$). The voltage at the coupling capacitor is then approximately the output voltage $U_{\text{out}}(t)$ of the voltage amplifier. Let us also again assume that the voltage induced into the tickler coil from the tuned circuit coil is much smaller than the AC output voltage $U_{\text{out}}(t)$ of the amplifier. The current $I_{\text{f}}(t)$ in the tickler coil is given by

$$I_{\text{f}}(t) = \dot{Q}_{\text{f}}(t)$$

where $\dot{Q}_{\text{f}}(t)$ is the first derivative of the charge of the coupling capacitor with respect to time. Since $Q_{\text{f}}(t) = C_{\text{f}} U_{\text{out}}(t)$ (remember that the voltage at the coupling capacitor is $U_{\text{out}}(t)$ under the previous assumptions) and $U_{\text{out}}(t) = \alpha U_C(t)$ we get

$$I_\mathrm{f}(t) = C_\mathrm{f}\alpha\dot{U}_C(t)$$

and with $U_C(t) = Q(t)/C$ for the capacitor of the tuned circuit we finally arrive at

$$I_\mathrm{f}(t) = \alpha\frac{C_\mathrm{f}}{C}I(t)$$

where we have used that the current $I(t)$ and the capacitor charge $Q(t)$ in the tuned circuit are related by $\dot{Q}(t) = I(t)$. Differentiating the above expression twice with respect to time yields

$$\ddot{I}_\mathrm{f}(t) = \alpha\frac{C_\mathrm{f}}{C}\ddot{I}(t)$$

which, again, can readily be substituted into (1). Sorting the left hand side of the resulting equation by $I(t)$ and it's derivatives now gives

$$\boxed{\left(L - \frac{nL_\mathrm{f}C_\mathrm{f}\alpha k}{n_\mathrm{f}C}\right)\ddot{I}(t) + R\dot{I}(t) + \frac{1}{C}I(t) = \dot{U}_0(t)}$$

This result is substantially different from the previous results. The physical loss resistance R stays untouched. Instead, the physical inductance L of the tuned circuit is replaced by a virtual inductance

$$\tilde{L} = L - \frac{nL_\mathrm{f}C_\mathrm{f}\alpha k}{n_\mathrm{f}C}$$

Under these circumstances, the circuit will not function as a regenerative amplifier or oscillator. In a frequency domain approach this can be understood as the coupling capacitor introducing a $+90°$ phase shift in the feedback, thereby breaking the phase relation between output and input necessary for regeneration and/or oscillations.

Dominant Tickler Coil Reactance

Finally, we will look at the case where the absolute value of the reactance of the tickler coil is much bigger than the output impedance of the voltage amplifier and the absolute value of the reactance of the coupling capacitor ($\omega L_\mathrm{f} \gg R_\mathrm{out}$ and $\omega L_\mathrm{f} \gg 1/\omega C_\mathrm{f}$). This is the case when the voltage amplifier has a very low output impedance and the coupling capacitor is made sufficiently large. Again, we assume that the voltage induced into the tickler coil from the tuned circuit coil is much smaller than the AC output voltage $U_\mathrm{out}(t)$ of the amplifier. The voltage applied to the tickler coil is then approximately the output voltage $U_\mathrm{out}(t)$ of the voltage amplifier and by virtue of the laws of electromagnetic induction [3] we have

$$L_\mathrm{f}\dot{I}_\mathrm{f}(t) = U_\mathrm{out}(t)$$

Making once more use of $U_\mathrm{out}(t) = \alpha U_C(t)$, $U_C(t) = Q(t)/C$ and $\dot{Q}(t) = I(t)$ we get

$$\ddot{I}_\mathrm{f}(t) = \frac{\alpha}{L_\mathrm{f}C} I(t)$$

which is again substituted into (1) and after sorting the left hand side of the resulting equation produces

$$\boxed{L\ddot{I}(t) + R\dot{I}(t) + \frac{1}{C}\left(1 - \frac{n\alpha k}{n_\mathrm{f}}\right) I(t) = \dot{U}_0(t)}$$

This time, the physical capacitance C of the tuned circuit gets replaced by a virtual capacitance

$$\tilde{C} = \left(1 - \frac{n\alpha k}{n_\mathrm{f}}\right)^{-1}$$

while, again, the physical loss resistance remains untouched. Under these circumstances, the circuit will also not function as a regenerative amplifier or oscillator. Again, in a frequency domain approach this can be understood as the tickler coil introducing a $-90°$ phase shift in the feedback, thereby again breaking the phase relation between output and input necessary for regeneration and/or oscillations.

Summary And Conclusions

It has become obvious that different types of feedback will alter the behavior of the tuned circuit in different ways. For the purpose of building regenerative amplifiers or oscillators, the only desired change in tuned circuit behavior is where the physical loss resistance is replaced by a lower virtual loss resistance. Any other change (virtual inductance, virtual capacitance) will only lead to detuning and is an undesired side-effect. From this point of view, putting the tickler coil in the plate circuit of a pentode is the optimal solution since this is equivalent to providing feedback by a controlled current source, which has been shown to affect only the virtual loss resistance and leave the other parameters of the tuned circuit as they are. If instead a voltage amplifier is used to drive the tickler coil, it needs to have an output impedance *above* a certain minimum that is determined by the capacitance of the coupling capacitor and the inductance of the tickler coil. This is in contrast to many applications of voltage amplifiers, where a low output impedance is desired.

Appendix A: Functions, Variables And Constants

Derivatives of a function with respect to time
In this paper, the general convention of representing differentiation with respect to time by a dot over the function is adopted. Hence,

$$\dot{f}(t) = \frac{d}{dt} f(t)$$

and

$$\ddot{f}(t) = \frac{d^2}{dt^2} f(t)$$

List of selected variables and constants

I_f: Current in the tickler coil

I: Current in the tuned circuit

U_C: Voltage at the capacitor of the tuned circuit

U_L: Voltage at the inductor (coil) of the tuned circuit

U_R: Voltage at the loss resistance of the tuned circuit

U_f: Feedback voltage induced into the tuned circuit by the tickler coil

U_0: Driving voltage of the tuned circuit ($U_0 \equiv 0$ for oscillator circuits)

L: Inductance of the coil of the tuned circuit

$\tilde{L}$: Virtual inductance of the tuned circuit

L_f: Inductance of the tickler coil

C: Capacitance of the capacitor of the tuned circuit

Q: Charge of the capacitor of the tuned circuit

C_f: Capacitance of the coupling capacitor

$\tilde{C}$: Virtual capacitance of the tuned circuit

Q_f: Charge of the coupling capacitor

R: Physical loss resistance of the tuned circuit

$\tilde{R}$: Virtual loss resistance of the tuned circuit

k: Inductive coupling constant between the tickler and the tuned circuit coil

n: Number of turns of the coil of the tuned circuit

n_{f}: Number of turns of the tickler coil

β: Transconductance of the voltage controlled current source

α: Open circuit amplification factor of the voltage amplifier

U_{out}: Open circuit output voltage of the voltage amplifier

R_{out}: Output impedance of the voltage amplifier

R_{plate}: Resistance of the plate resistor

References

[1] Jochen Bauer, http://www.radiomuseum.org/forumdata/users/133/PDF/RegenerationLC.pdf

[2] Meyberg, Vachenauer, *Höhere Mathematik II S. 43, Springer*, 1991

[3] http://en.wikipedia.org/wiki/Inductance